Factors Influencing Heat Transfer to the Pressure Surfaces of Gas Turbine Blades

B. W. Martin* and A. Brown*

It is suggested that heat transfer through the laminar boundary layer flowing over the concave pressure surface of a turbine blade is strongly influenced by the presence of Taylor-Goertler vortices, as well as by mainstream turbulence. Transition occurs when these factors in concert outweigh the tendency of the boundary layer to remain laminar in the favourable pressure gradients characteristic of flow over pressure surfaces.

NOTATION

a, b	coefficients in eq. (4)
c	coefficients in eq. (6)
G	Goertler number
G_{δ_2}	Goertler number based on boundary-layer momentum thickness $[= (U_\infty \delta_2/\nu)^2 \delta_2/r_c]$
K	velocity gradient factor $[= (\nu/U_\infty^2)\, dU_\infty/dx]$
L	length of surface
M	Mach number
m, n, p	exponents in eq. (4)
Pr	Prandtl number
r_c	radius of curvature of blade pressure surface
Re_x	length Reynolds number $[= U_\infty x/\nu]$
Re_s	trailing-edge Reynolds number $[= U_2 s/\nu]$
Re_{δ_2}	momentum-thickness Reynolds number $[= U_\infty \delta_2/\nu]$
s	pressure surface length
St	Stanton number
U_0	initial mainstream velocity
U_2	trailing-edge mainstream velocity
U_∞	local mainstream velocity
U'_∞/U_∞	mainstream turbulence intensity
u	mainstream velocity gradient $[= dU_\infty/dx]$
x	distance downstream from blade leading edge
y	distance normal to blade surface
z	spanwise distance along blade
δ	boundary-layer thickness
δ_2	boundary-layer momentum thickness
Λ	modified Pohlhausen parameter $[= (\delta_2^2/\nu)\, dU_\infty/dx]$
λ	wavelength of Taylor-Goertler vortices
ν	kinematic viscosity

INTRODUCTION

In a paper (**1**) presented to the 1979 ASME Gas Turbine and Solar Energy Conference, the authors re-assessed by application to recently-published turbine-blade heat-transfer measurements the mainly-empirical criteria currently used to predict boundary-layer behaviour under the combined influence of velocity gradient factor $K = (\nu/U_\infty^2)\, dU_\infty/dx$ and significant mainstream turbulence. The boundary-layer phenomena of interest to the blade designer include separation, laminarization and the onset and extent of transition, and the available evidence indicated that, under the conditions experienced in gas turbine engines, the scale and frequency of mainstream turbulence might be as important as its intensity in determining local heat transfer coefficients round the blades.

As has been demonstrated by the authors in the above, and earlier papers (**2, 3**), in the presence of significant mainstream turbulence the heat transfer on the pressure surface of a turbine blade (where the pressure gradients are mostly strongly favourable) is generally even less well predicted than on the suction surface. This can be attributed, not only to the inadequacies of such transition indicators as those of Seyb (**4**) for its onset, and Dhawan and Narasimha (**5**) for its extent (these are among the best available) but also to our lack of understanding of the factors which bring about higher levels of upstream heat transfer, with substantial chordwise fluctuations, than those associated with a laminar boundary layer.

Walker and Markland (**6**) and Dunham and Edwards (**7**) both describe the expected laminar boundary layer on the pressure surface in cascade tests as 'transitional with occasional bursts of turbulence' in an attempt to explain the marked fluctuations in local heat transfer which they observed. Dunham and Edwards (**7**) also found the surprisingly high heat transfer on the pressure surface to increase further with mainstream Mach number M up to 0·9. The latter increase may be associated with the scale of mainstream turbulence, as suggested by Brown and Martin (**1**). It is then tempting to ascribe the increased heat transfer above that for a laminar boundary layer (even at low M) to the influence of mainstream turbulence intensity U'_∞/U_∞, as in the case of the laminar boundary layer on the suction surface in the presence of a favourable pressure gradient.

Buyuktur, Kestin, and Maeder (**8**) and Brown and Burton (**9**) observed an approximately linear dependence of laminar heat transfer on U'_∞/U_∞; the latter workers correlated suction surface measurements on a blade of prescribed velocity distribution (PVD) at low M by a multiplying factor to the standard laminar boundary-layer heat-transfer relation for $U'_\infty/U_\infty \leq 0{\cdot}09$. Application of this correlation by Brown and Martin (**1**) to the aerodynamic measurements of Martin, Brown and Garrett (**3**) for $0{\cdot}04 \leq U'_\infty/U_\infty \leq 0{\cdot}09$ at high

* Department of Mechanical Engineering and Engineering Production, University of Wales Institute of Science and Technology, Cardiff, CF1 3NU, U.K.

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subsonic speeds not only overpredicted laminar boundary-layer heat transfer on both the suction and pressure surfaces (by amounts which increased with U'_{∞}/U_{∞}), but also failed to predict the marked chordwise heat transfer fluctuations along the pressure surface, also observed (as previously noted) by Walker and Markland (**6**) and Dunham and Edwards (**7**).

Since in the laminar regions on suction surfaces no similar heat-transfer fluctuations have been reported, they cannot readily be attributed to any of the characteristics of mainstream turbulence, or in any obvious way to curvature effects on the boundary-layer flow. However, there remains the possibility that they result from the Taylor–Goertler vortices, also postulated by Forest (**10**), caused by three-dimensional disturbances within the boundary-layer flow along the concave pressure surface (as shown in Fig. 1). Such vortices, first predicted by Goertler (**11**), which grow with distance in the direction of mainstream flow, are formed by the instability which occurs when the centrifugal forces acting on the fluid become so large that the radial pressure gradient and viscous forces can no longer damp out small disturbances. The vortices also give rise to turbulence when the entire range of wavelengths experience amplification and it would appear from the work of Liepmann (**12**, **13**) that on concave surfaces of large curvature, the transition to turbulence of the laminar boundary layer is mainly due to the growth of Taylor–Goertler vorticity.

The purpose of this paper is, therefore, to establish the extent to which Taylor–Goertler vortices are likely to occur over the pressure surface of a turbine blade for the ranges of mainstream velocity distributions and length Reynolds number Re_x appropriate to engine operating conditions. Furthermore, having shown that such vortices must be present over much of the surface, to attempt to assess how this form of instability interacts with mainstream turbulence and the laminarization or reverse transition associated with values of $K \geq 2{\cdot}5 \times 10^{-6}$ in determining the onset and extent of transition on the pressure surface of a PVD blade. The heat-transfer measurements of McCormack, Welker, and Kelleher (**14**) over a concave wall in the presence of cellular vortices seem to offer a possible explanation of the observed fluctuations in (and levels of) local heat-transfer coefficients along the pressure surfaces of turbine blades.

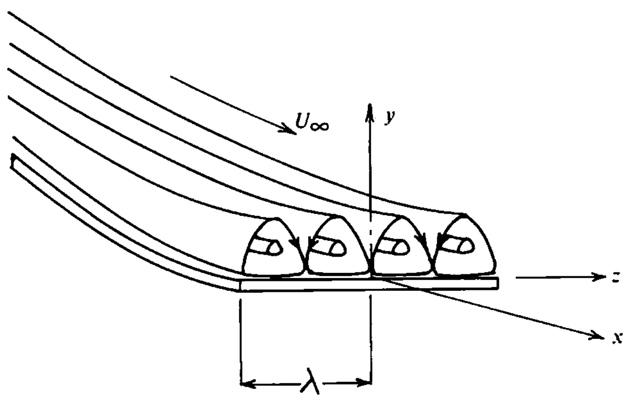

Fig. 1. Diagrammatic representation of Taylor–Goertler vortices on a concave surface

TAYLOR–GOERTLER VORTICES AND THEIR EFFECT ON TRANSITION

As already stated, Goertler (**11**) was the first to predict the formation of vortices within the laminar boundary layer over a concave surface. He analyzed the development of such secondary flow with time, postulating a small three-dimensional disturbance superimposed on the base flow in an attempt to specify the neutral stability condition. He found that at the onset of vortex motion, which occurs for all values of wavelength λ, the characteristic parameter now known as the Goertler number G has a certain positive value; this is determined by the wave number $2\pi\delta/\lambda$ and passes through a minimum at $2\pi\delta/\lambda = 1{\cdot}5$, where δ is the boundary-layer thickness. At this stage of neutral stability the amplification factor is zero. For a given wave number, the value of G subsequently increases with the amplification factor.

The later extensive numerical analysis of Smith (**15**), who considered disturbances which grow with distance, predicted lower values of G than Goertler for the neutral stability condition over much of the range of possible wave numbers. Basing G on boundary-layer momentum thickness δ_2, Smith showed that vortex formation occurs only if

$$G_{\delta_2} = \left(\frac{U_\infty \delta_2}{\nu}\right)^2 \frac{\delta_2}{r_c} \geq 0{\cdot}09 \qquad (1)$$

which minimum corresponds to a wave number of about 0·5. According to Schlichting (**16**), Smith's predictions have received theoretical and experimental confirmation through measurements on bodies with external concave walls placed in a stream.

More recent work has been mainly theoretical. Herbert (**17**) compared the various approaches for investigating the linear stability of a laminar boundary layer along a concave wall with respect to Taylor–Goertler vortices. Kahawita and Meroney (**18**) found that the effect of heating was to stabilize the flow to disturbances of long lateral wavelength but to destabilize it to disturbances of short wavelength. Kobayash and Kohama (**19**) predicted an increase in G for neutral stability of about 60 per cent by increasing M from zero to five in the case of a thermally-insulated wall. Though these predictions may imply some increase in the minimum value of G for neutral stability under turbine-blade operating conditions, they have yet to be verified experimentally and in the consideration of flow over the pressure surfaces described below, Smith's (**15**) lower limit given by relation (1) is used.

The only measurements of the effect of longitudinal vortices on heat transfer through the boundary layer of which the authors are aware are those already mentioned of McCormack, Welker, and Kelleher (**14**), which indicate marked periodic heat-transfer fluctuations in the spanwise direction and a significant increase in Nusselt number in the presence of vortices. These are discussed in detail below.

The transition to turbulence in the boundary layer flowing along a concave wall occurs a considerable distance downstream of the stability limit owing to the finite time and distance required for amplification of Taylor–Goertler vortices. The measurements of Liepmann (**12**, **13**) suggest that transition takes place when G_{δ_2} exceeds

49 but Dryden (**20**) states that the value ranges from 81 for very low U'_∞/U_∞ to 36 for U'_∞/U_∞ of 0·003, and evidently diminishes rapidly with increasing mainstream turbulence intensity. Liepmann's measurements of transition on concave surfaces for very low U'_∞/U_∞ are correlated over the range $0 \leq \delta_2/r_c \leq 1{\cdot}8 \times 10^{-4}$ by

$$\mathrm{Re}_{\delta_2} = \left[\frac{G_{\delta_2}}{(\delta_2/r_c)}\right]^{1/2} = -21{\cdot}8 \times 10^5 \frac{\delta_2}{r_c} + 953 \quad (2)$$

However, this cannot reasonably be expected to hold good for the range $0{\cdot}04 \leq U'_\infty/U_\infty \leq 0{\cdot}09$ considered below in the discussion of the influence of Taylor–Goertler vortices on transition on the pressure surface of a PVD blade; even less so under gas-turbine engine operating conditions where U'_∞/U_∞ is likely to be even greater, perhaps between 0·12 and 0·15. Based on somewhat slender evidence, Forest (**10**) assumes an exponential relation between G_{δ_2} and U'_∞/U_∞ given by

$$G_{\delta_2} = 81 \exp\left(-34{\cdot}6 \frac{U'_\infty}{U_\infty}\right) \quad (3)$$

which, as will be seen, correctly predicts the transition value for G_{δ_2} of about 8·9 (and $\mathrm{Re}_{\delta_2} = 193$) for the PVD blade considered below, but only for $U'_\infty U_\infty = 0{\cdot}064$, which is in the middle of the experimental range $0{\cdot}04 \leq U'_\infty/U_\infty \leq 0{\cdot}09$. Also, although eq. (3) agrees with Dryden's (**20**) finding of $G_{\delta_2} = 81$ for transition at very low U'_∞/U_∞, clearly it overpredicts the value of G_{δ_2} for transition at $U'_\infty/U_\infty = 0{\cdot}003$, giving a value of 73 as compared to Dryden's measured value of 36.

GOERTLER NUMBER FOR SOME FREESTREAM VELOCITY DISTRIBUTIONS

Many freestream velocity distributions are described by the general equation

$$U_\infty = U_0 x^m (a + bx^n)^p \quad (4)$$

where the flows are distinguished by the values of a, b, m, n, and p for a given initial velocity U_0. For potential or stagnation flow $p = 0$ and eq. (4) simplifies to

$$U_\infty = U_0 x^m \quad (5)$$

whereas for Hiemenz flow as defined by Buyuktur, Kestin, and Maeder (**8**) $m = 0$, $a = n = p = 1$, $b = c/L$ and eq. (4) becomes

$$U_\infty = U_0\left(1 + \frac{cx}{L}\right) \quad (6)$$

where c is a constant and L is the length of surface over which the fluid flows. The special case of $c = 1$ was that selected by Buyuktur *et al.* (**8**) to measure the combined influence of pressure gradient and freestream turbulence on heat transfer from a plate of length L. When $c = 0$ in eq. (6) or $m = 0$ in eq. (5) the uniform freestream velocity is that for the Blasius flow; if $c < 0$ the resultant adverse pressure gradient yields flow separation when $cx/L = -0{\cdot}12$ according to Curle and Davies (**21**). The Curle and Davies laminar boundary-layer separation criterion is in terms of the modified Pohlhausen (**22**) parameter $\Lambda = -0{\cdot}09$.

A further important case of practical application is that of uniform velocity gradient. In this case, if u is the uniform velocity gradient, eq. (4) with $m = 0$, $a = n = p = 1$ and $b = u/U_0$ gives

$$U_\infty = U_0\left(1 + \frac{ux}{U_0}\right) \quad (7)$$

Equations (6) and (7) are identical if $cx/L = ux/U_0$. Finally, the case of constant velocity gradient factor K is of interest for comparative purposes. Brown and Martin (**23**) have previously shown that K is a useful parameter for predicting laminar boundary-layer behaviour and for comparison of the boundary layers associated with a range of freestream velocity distributions. For this case $m = 0$, $a = n = 1$, $p = -1$, and $b = -U_0K/\nu$ in eq. (4) giving

$$U_\infty = U_0\bigg/\left(1 - \frac{U_0Kx}{\nu}\right)$$
$$= U_0\left(1 + \frac{U_\infty Kx}{\nu}\right) = U_0(1 + K\,\mathrm{Re}_x) \quad (8)$$

where the kinematic viscosity ν is assumed constant and $\mathrm{Re}_x = U_\infty x/\nu$. Velocity distributions other than those described above are possible but it is suggested that the distributions represented by eqs. (5), (6), (7), and (8) are sufficient to cover all practical cases. For example, any freestream velocity distribution could be described by a series of interconnected constant velocity gradient distributions; the accuracy depends on the number of subdivisions into which the original freestream velocity distribution is divided.

In order to determine expressions for the Goertler number $G_{\delta_2}[= (U_\infty/\nu)^2\delta_2^3/r_c]$ for a given freestream velocity distribution it is necessary to obtain an expression relating the laminar boundary-layer momentum thickness δ_2 to the freestream velocity U_∞. Thwaites (**24**) obtained such an expression by noting that the two-dimensional laminar boundary-layer momentum-integral equation can be closely approximated by

$$\frac{U_\infty}{\nu}\frac{d\delta_2^2}{dx} = 0{\cdot}45 - 6\Lambda \quad (9)$$

which on integration gives

$$\delta_2^2 = \frac{0{\cdot}45\nu}{U_\infty^6}\int_0^x U_\infty^5\,dx \quad (10)$$

Thwaites found that eq. (9) had a high degree of accuracy when compared with measurements of the principal characteristics of laminar boundary layers. Other workers have found eqs (9) and (10) useful in examining laminar boundary-layer behaviour [see Brown and Martin (**23**)]. Substituting from eqs. (5), (6), (7), and (8) into eq. (10) in turn, corresponding expressions for the Goertler number are obtained as follows

$$G_{\delta_2}\frac{r_c}{x\,\mathrm{Re}_x^{0{\cdot}5}} = \left(\frac{0{\cdot}45}{5m + 1}\right)^{1{\cdot}5} \quad (11)$$

$$G_{\delta_2}\frac{r_c}{x\,\mathrm{Re}_x^{0{\cdot}5}} = \left\{0{\cdot}075\left(1 + \frac{L}{cx}\right)\left[1 - \left(1 + \frac{cx}{L}\right)^{-6}\right]\right\}^{1{\cdot}5} \quad (12)$$

$$G_{\delta_2}\frac{r_c}{x\,\mathrm{Re}_x^{0{\cdot}5}} = \left\{0{\cdot}075\left(1 + \frac{U_0}{ux}\right)\left[1 - \left(1 + \frac{ux}{U_0}\right)^{-6}\right]\right\}^{1{\cdot}5} \quad (13)$$

and

$$G_{\delta_2}\frac{r_c}{x\,\mathrm{Re}_x^{0\cdot5}} = \left\{\frac{0\cdot1125}{K\,\mathrm{Re}_x}\left[1-(1+K\,\mathrm{Re}_x)^{-4}\right]\right\}^{1\cdot5} \tag{14}$$

for potential or stagnation flow, Hiemenz flow, uniform velocity gradient flow, and constant velocity gradient factor flow, respectively.

Examination of eqs. (11)–(14) shows that Goertler vortices can exist in favourable, zero, and adverse pressure gradients, but there are limits to the adverse pressure gradient at which the Goertler number becomes imaginary. These limits occur at $m < -0{\cdot}2$; $-1 > cx/L > -2$; $-1 > ux/U_0 > -2$, and $-2 > K\,\mathrm{Re}_x > -\infty$, respectively, for the four flows under consideration. The imaginary values of Goertler number although mathematically correct are physically meaningless. In all cases the laminar boundary layer separates before the 'imaginary' limits are reached. According to Curle and Davies (**21**) laminar boundary-layer separation occurs when $\Lambda = -0{\cdot}09$, and for the four flows this occurs when $m = -0{\cdot}1$, $cx/L = -0{\cdot}123$, $ux/U_0 = -0{\cdot}123$, and $K\,\mathrm{Re}_x = -0{\cdot}137$, respectively. Goertler vortices clearly cannot exist in adverse pressure gradients beyond the point of laminar separation.

Equations (6) and (7) and eqs. (12) and (13), respectively, are identical when $cx/L = ux/U_0$, and so for convenience only three flows will be considered in the remainder of this article. The flows are represented by eqs. (5), (7), and (8) with values for the Goertler number obtained from eqs. (11), (13), and (14). The velocity gradient factor K may be used to place the three velocity distributions and the corresponding expressions for Goertler number on a common basis. Equations (11), (13), and (14) and the practical range of $K\,\mathrm{Re}_x$ from laminar boundary-layer separation to an infinite favourable velocity gradient are modified, respectively, to the following

$$G_{\delta_2}\frac{r_c}{x\,\mathrm{Re}_x^{0\cdot5}} = \left(\frac{0\cdot45}{5K\,\mathrm{Re}_x+1}\right)^{1\cdot5}, \qquad -0\cdot1 \le K\,\mathrm{Re}_x \le \infty \tag{15}$$

$$G_{\delta_2}\frac{r_c}{x\,\mathrm{Re}_x^{0\cdot5}} = \left\{\frac{0\cdot075}{K\,\mathrm{Re}_x}\left[1-(1-K\,\mathrm{Re}_x)^{6}\right]\right\}^{1\cdot5}, \qquad -0\cdot140 \le K\,\mathrm{Re}_x \le 1 \tag{16}$$

and

$$G_{\delta_2}\frac{r_c}{x\,\mathrm{Re}_x^{0\cdot5}} = \left\{\frac{0\cdot1125}{K\,\mathrm{Re}_x}\left[1-(1+K\,\mathrm{Re}_x)^{-4}\right]\right\}^{1\cdot5}, \qquad -0\cdot137 \le K\,\mathrm{Re}_x \le \infty \tag{17}$$

Equations (15), (16), and (17) are plotted in Fig. 2 from which it can be seen that the variation in $G_{\delta_2} r_c/(x\,\mathrm{Re}_x^{0\cdot5})$ between the three flows is small for $K\,\mathrm{Re}_x < 1$. The constant velocity gradient case falls between the other two and in this case $K\,\mathrm{Re}_x = 1$ corresponds to an infinite favourable velocity gradient. In view of these findings, and their relevance to pressure surfaces of gas turbine blades, it was decided to concentrate on the constant velocity gradient case which is replotted for constant values of K in Figs. 3 and 4 in terms of $G_{\delta_2} r_c/x$ versus Re_x.

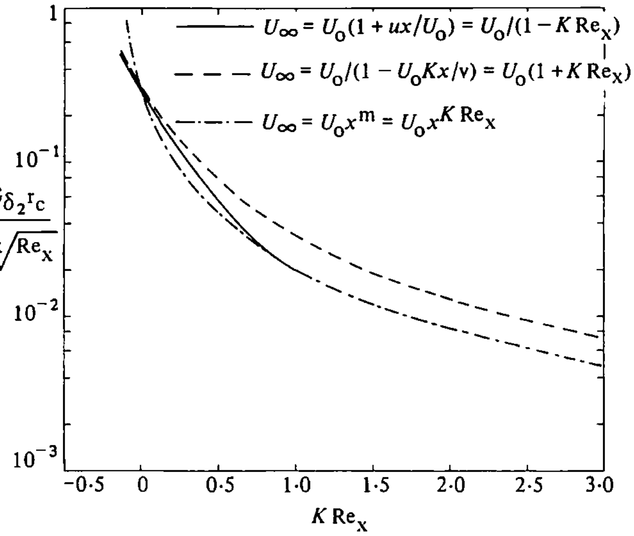

Fig. 2. Variation of $G_{\delta_2} r_c/(x\,\mathrm{Re}_x^{1/2})$ with $K\,\mathrm{Re}_x$ for potential flow, uniform velocity gradient flow, and constant velocity gradient factor flow

Figure 3 is a useful plot of $G_{\delta_2} r_c/x$, velocity gradient factor and Reynolds number based on eq. (16) for the flow case of constant favourable velocity gradient. Launder (**25, 26**) showed, and it has since been confirmed by the heat-transfer measurements of Kearney, Kays, and Moffatt (**27**), Moretti and Kays (**28**), Back and Seban (**29**), and Slanciauskas and Pedisius (**30**), that for $K \ge 2{\cdot}5 \times 10^{-6}$ the boundary-layer flow remains laminar or is laminarized. Thus if the condition of eq. (1), i.e., $G_{\delta_2} \ge 0{\cdot}09$ is met, Goertler vortices will exist for $K \ge 2{\cdot}5 \times 10^{-6}$, but it is possible that they will not exist for $K < 2{\cdot}5 \times 10^{-6}$ if the boundary-layer flow goes into transition and becomes turbulent, except perhaps in the laminar sublayer. Various boundary-layer transition predictors are available, but perhaps the most frequently used for the start of transition is that of Seyb (**4**) given by

$$\mathrm{Re}_{\delta_2} = \frac{1000}{(1 + 70U'_\infty/U_\infty)} + 10\left(\frac{\Lambda + 0\cdot09}{0\cdot0106 + 3\cdot6U'_\infty/U_\infty}\right)^{2\cdot62} \tag{18}$$

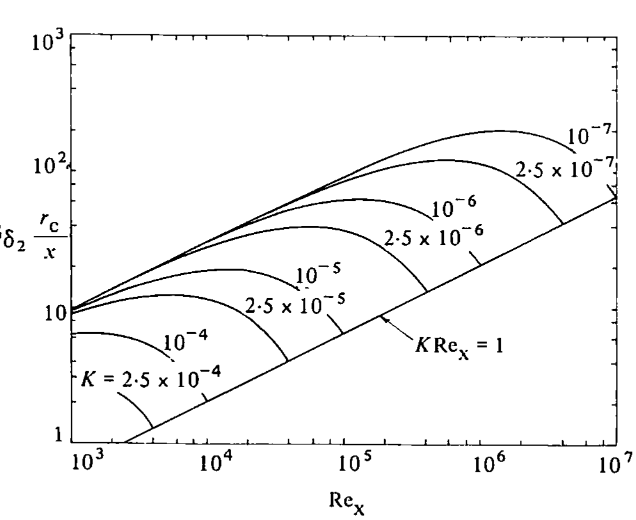

Fig. 3. Relation between $G_{\delta_2} r_c/x$ and Re_x at various positive K for uniform velocity gradient flow

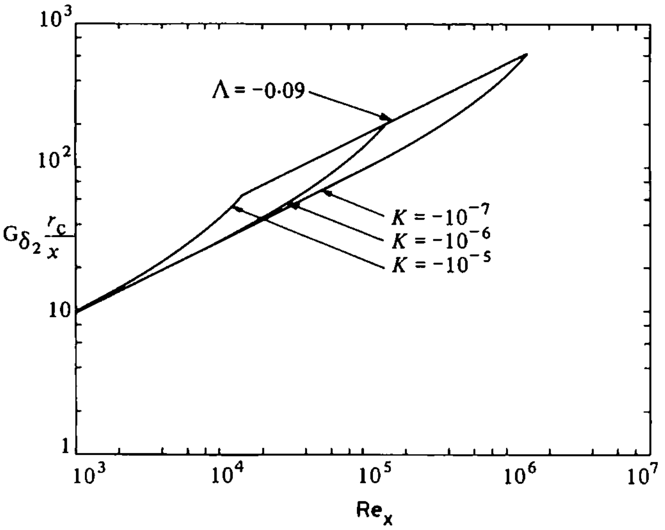

Fig. 4. Relation between $G_{\delta_2} r_c/x$ and Re_x at various negative K for uniform velocity gradient flow

where for a constant velocity gradient

$$\Lambda = 0{\cdot}075[1 - (1 - K\,Re_x)^6] \qquad (19)$$

Seyb suggested that eq. (18) was valid for $U'_\infty/U_\infty \leq 0{\cdot}04$. However, Brown and Burton (**9**) have shown that for low-speed flows Seyb's criterion can be extended to $U'_\infty/U_\infty \simeq 0{\cdot}06$. For fixed values of K ($< 2{\cdot}5 \times 10^{-6}$) and $U'_\infty/U_\infty (< 0{\cdot}06)$ it is possible through eqs. (18) and (19) and the definition of the Goertler number to obtain a plot of $G_{\delta_2} r_c/x$ versus Re_x for the start of boundary-layer transition according to Seyb in a favourable velocity gradient as shown in Fig. 5. Combining Figs. 3 and 5 gives the conditions for the presence of Goertler vortices in constant favourable velocity gradient flows. Consider $Re_x = 10^5$ and $K = 2{\cdot}5 \times 10^{-6}$, then from Fig. 3 $G_{\delta_2} r_c/x = 39$ and Goertler vortices exist provided $G_{\delta_2} > 0{\cdot}09$, according to eq. (1). However, from Fig. 5 for the same conditions the values of $G_{\delta_2} r_c/x$ for transition according to Seyb are 820, 176, and 67 for $U'_\infty/U_\infty = 0{\cdot}02$, $0{\cdot}04$, and $0{\cdot}06$, respectively. Thus, for these three conditions the boundary-layer flow would be laminar and Goertler vortices could exist. Consider $Re_x = 10^5$ and $K = 2{\cdot}5 \times 10^{-7}$, then from Fig. 3, $G_{\delta_2} r_c/x = 87$ and from Fig. 5, $G_{\delta_2} r_c/x = 645$, 162, and 65 for $U'_\infty/U_\infty = 0{\cdot}02$, $0{\cdot}04$, and $0{\cdot}06$, respectively; thus Goertler vortices could exist for $U'_\infty/U_\infty = 0{\cdot}02$ and $0{\cdot}04$, but for $U'_\infty/U_\infty = 0{\cdot}06$ (according to Seyb) transition has taken place, and if Goertler vortices exist they can only be in the laminar sublayer.

It is possible to have Goertler vortices in adverse velocity gradient flows but the limited range over which they can exist is demonstrated by Fig. 4 and the region $K\,Re_x < 0$ of Fig. 2. Although adverse velocity gradients do occasionally exist on gas turbine blade pressure surfaces they are of necessity small. For the remainder of this article only favourable velocity gradients will be considered with reference to pressure surfaces of gas turbine blades.

TAYLOR-GOERTLER VORTICITY ON TURBINE BLADE PRESSURE SURFACES

In this section consideration is given to the cascade heat-transfer measurements of Martin *et al.* (**3**) for trailing-edge Reynolds numbers of 8×10^5 and $8{\cdot}8 \times 10^5$ on a turbine blade of 50 mm surface length s. The radius of curvature r_c of the pressure surface was nearly constant at 80 mm giving $0 \leq x/r_c \leq 0{\cdot}625$. [Assuming that the pressure surface of a typical blade can be approximated by a single circular arc, it follows that for a very large turning angle of 90°, $0 \leq x/r_c \leq \pi/2$. Turning angles are usually about 60°, in which case $0 \leq x/r_c \leq \pi/3$. The practical working range is, therefore, roughly $0 \leq x/r_c \leq 1$. Figure 6 shows the variation of G_{δ_2} with

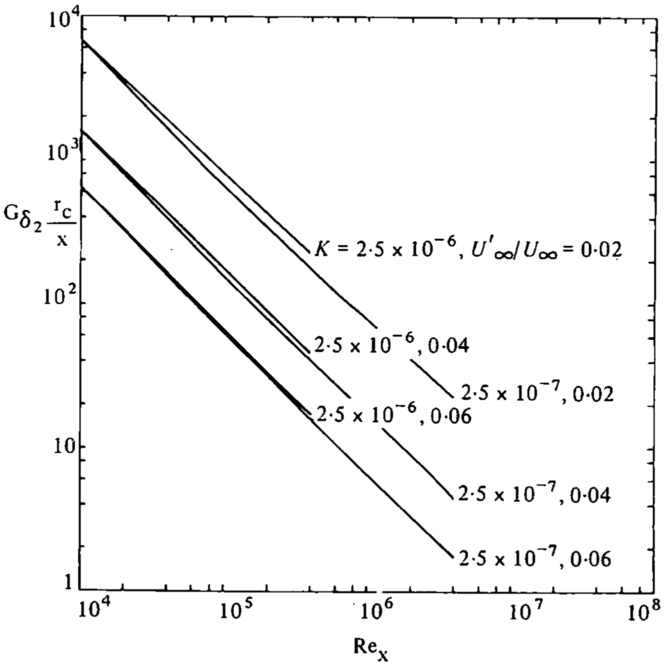

Fig. 5. Relation between the Seyb (**4**) transition criterion (in terms of $G_{\delta_2} r_c/x$) and Re_x for various positive K and U'_∞/U_∞ in uniform velocity gradient flow

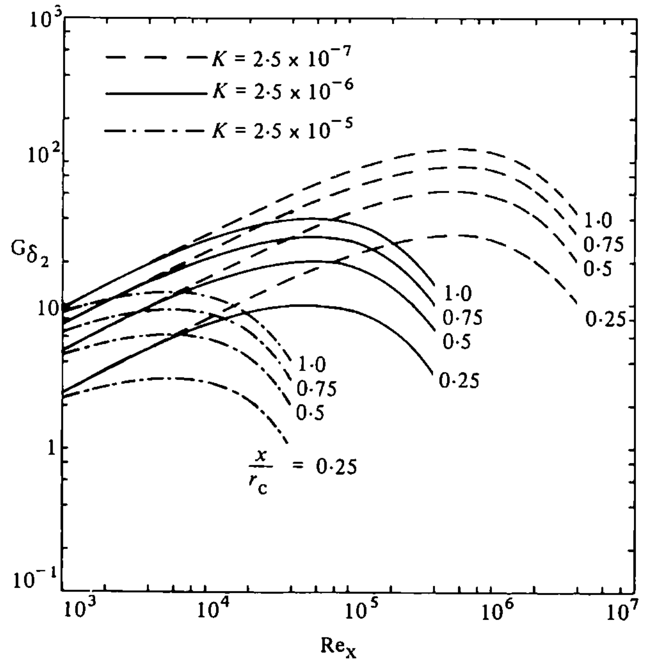

Fig. 6. Variation of G_{δ_2} with Re_x for positive K as a function of x/r_c in uniform velocity gradient flow

Re_x for this range of x/r_c and for three values of K. The latter are typical of pressure surfaces of turbine blades but also center on the Launder (**25**, **26**) laminarization criterion of $K \geq 2{\cdot}5 \times 10^{-6}$]. The mainstream velocity distribution on the pressure surface is described by

$$U_\infty = 3{\cdot}78 U_2 \frac{x}{s}, \qquad 0 \leq \frac{x}{s} \leq 0{\cdot}107 \qquad (20)$$

and

$$U_\infty = 0{\cdot}328 U_2 \left(1 + 2{\cdot}04 \frac{x}{s}\right), \qquad 0{\cdot}107 \leq \frac{x}{s} \leq 1 \quad (21)$$

Substitution of eqs. (20) and (21) in eq. (13) gives

$$G_{\delta_2} \frac{r_c}{x} = 0{\cdot}0205 \, Re_x^{1/2}, \qquad 0 \leq \frac{x}{s} \leq 0{\cdot}107 \qquad (22)$$

and

$$G_{\delta_2} \frac{r_c}{x} = 0{\cdot}0071 \, Re_x^{1/2} \left(\frac{\{[1 + 2{\cdot}04(x/s)]^6 - 1\}}{(x/s)[1 + 2{\cdot}04(x/s)]^5} \right)^{3/2}$$

$$0{\cdot}107 \leq \frac{x}{s} \leq 1 \quad (23)$$

Equations (22) and (23) are plotted in Fig. 7 both in the form $G_{\delta_2}(r_c/x)$ and G_{δ_2} against Re_x for $Re_s = 8 \times 10^5$. It should be noted that

$$K \, Re_x = 1, \qquad 0 \leq \frac{x}{s} \leq 0{\cdot}107 \qquad (24)$$

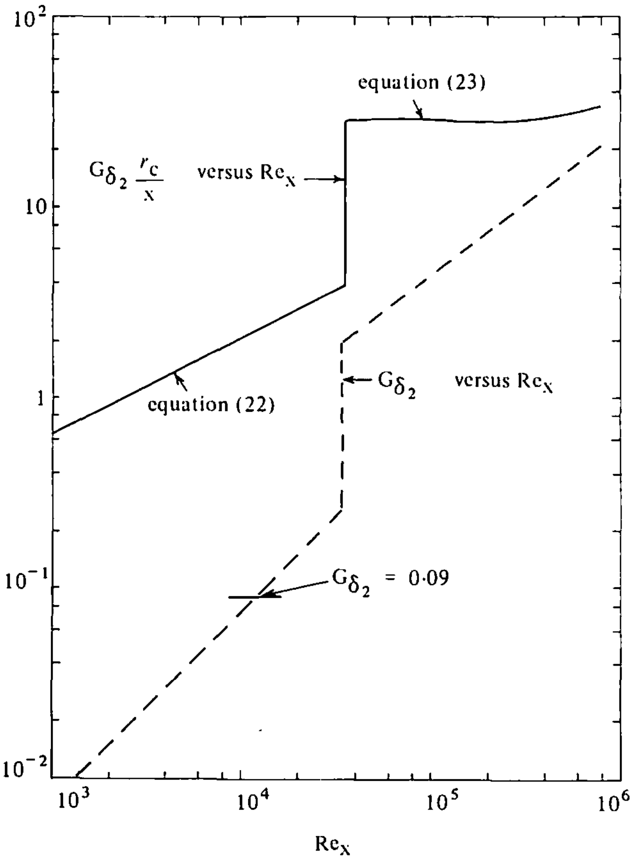

Fig. 7. Variation of $G_{\delta_2} r_c/x$ and G_{δ_2} with Re_x for flow over the pressure surface of the blade of Martin *et al.* (3)

and

$$K \, Re_x = \frac{2{\cdot}04(x/s)}{[1 + 2{\cdot}04(x/s)]}, \qquad 0{\cdot}107 \leq \frac{x}{s} \leq 1 \quad (25)$$

In Fig. 7 the step increases in $G_{\delta_2} r_c/x$ and G_{δ_2} arise from the change in velocity gradient at $x/s = 0{\cdot}107$. The assumed onset of vortex motion at $G_{\delta_2} = 0{\cdot}09$ corresponds to $Re_x = 1{\cdot}1 \times 10^4$ and $x/s = 0{\cdot}06$ (as indicated in Table 1). This is well within the range of eq. (20). Table 1 also includes the values taken from Fig. 7 of G_{δ_2}

Table 1

Observed values of Goertler number and associated parameters

	Onset of vortex motion	Trailing edge	Mid-chord
x/s	0·06	1·0	0·5
r_c/x	27·1	1·6	3·2
Re_x	$1{\cdot}1 \times 10^4$	8×10^5	$2{\cdot}65 \times 10^5$
K	$9{\cdot}1 \times 10^{-5}$	$8{\cdot}4 \times 10^{-7}$	$1{\cdot}91 \times 10^{-6}$
G_{δ_2}	0·09	20·6	8·97
$G_{\delta_2} r_c/x$	2·43	33·0	28·7

and $G_{\delta_2} r_c/x$ at the blade trailing edge and at mid-chord (where $x/s = 0{\cdot}5$) together with the corresponding values of K and Re_x. The latter are used in conjunction with Fig. 5 to determine the transition values of G_{δ_2} and $G_{\delta_2} r_c/x$ presented in Table 2, based on the Seyb (**4**) criterion, for three values of turbulence intensity within its validated range.

At the trailing edge, comparisons of $G_{\delta_2} r_c/x$ between Tables 1 and 2 indicate that Taylor–Goertler vortices should occur in a completely laminar boundary layer only for $U'_\infty/U_\infty = 0{\cdot}02$. At turbulence intensities of 0·04 and 0·06, if vortices occur at all, according to Seyb's criterion, they occur downstream of the onset of transition. Similarly at mid-chord, vortices should occur in a completely laminar boundary layer for turbulence intensities of 0·02 and 0·04, but downstream of transition for 0·06.

The pressure surface heat-transfer measurements of Martin *et al.* (**3**) reproduced in Fig. 8 suggest that for $Re_s = 8 \times 10^5$, boundary-layer transition in fact begins slightly upstream of $x/s = 0{\cdot}5$, when $Re_x = 2{\cdot}5 \times 10^5$ and $K \simeq 2 \times 10^{-6}$. This is so for all three turbulence intensities of 0·042, 0·057, and 0·083. To a good approximation based on the mid-chord values in Table 1, the observed transition value of $G_{\delta_2} r_c/x$ is, therefore, about 28·5. The corresponding value of G_{δ_2} is about 8·9; also for $0{\cdot}042 \leq U'_\infty/U_\infty \leq 0{\cdot}083$ at $K \simeq 2 \times 10^6$. This value should be compared with those of the Seyb criterion in Table 2 where $G_{\delta_2} = 20{\cdot}6$ for $U'_\infty/U_\infty = 0{\cdot}04$ and 7·97 for $U'_\infty/U_\infty = 0{\cdot}06$. These, of course, take no account of the effect of longitudinal vortices. The observed transition value of G_{δ_2} of 8·9 corresponds to $\delta_2/r_c = 2{\cdot}41 \times 10^{-4}$ and $Re_{\delta_2} = 193$, which is much less than the value of 428 derived for the same δ_2/r_c from a modest 34 per cent extrapolation of eq. (2).

As foreshadowed above, the transition values of Re_{δ_2} and G_{δ_2} for flow over concave surfaces in the presence of

Table 2

Transition values of Goertler number based on the Seyb criterion

Re_x	$8{\cdot}10^5$	$8{\cdot}10^5$	$8{\cdot}10^5$	$2{\cdot}65 \times 10^5$	$2{\cdot}65 \times 10^5$	$2{\cdot}65 \times 10^5$
K	$8{\cdot}4 \times 10^{-7}$	$8{\cdot}4 \times 10^{-7}$	$8{\cdot}4 \times 10^{-7}$	$1{\cdot}91 \times 10^{-6}$	$1{\cdot}91 \times 10^{-6}$	$1{\cdot}91 \times 10^{-6}$
U'_∞/U_∞	0·02	0·04	0·06	0·02	0·04	0·06
G_{δ_2}	61·9	13·4	5·3	93·8	20·6	7·97
$G_{\delta_2} r_c/x$	99·0	21·5	8·5	300	66·0	25·5

longitudinal vortices, therefore, diminish with increasing U'_∞/U_∞, though not apparently in accordance with eq. (3) due to Forest (**10**), which yields the observed transition value for G_{δ_2} of 8·9 only for $U'_\infty/U_\infty = 0{\cdot}064$, and predicts G_{δ_2} to be 19 and 4·6 for the extreme measured turbulence intensities of 0·042 and 0·083, respectively.

Mainstream turbulence intensity thus interacts with Taylor–Goertler vorticity to advance transition on concave surfaces according to a relationship which has yet to be reliably determined. It would seem, however, that the start of transition is controlled to an over-riding extent by the Launder (**25**, **26**) criterion of $K \geq 2{\cdot}5 \times 10^{-6}$ (to which the observed transition value of about 2×10^{-6} approximates) whereby the boundary layer remains laminar, or is relaminarized if this criterion is met. If not, then at least for the pressure-surface measurements considered, transition is determined by Taylor–Goertler vorticity (rather than by the Seyb criterion), but at a value of G_{δ_2} which is itself influenced by the level of mainstream turbulence intensity.

Figure 8 also shows that in the pre-transition range $0 \leq x/s \leq 0{\cdot}5$ the local heat transfer for $Re_s = 8{\cdot}8 \times 10^5$ exceeds that predicted by the standard laminar flat-plate correlation by a factor of about 2 and is almost insensitive to the range of U'_∞/U_∞ from 0·066 to 0·091. For $Re_s = 8 \times 10^5$ heat transfer in the region $0 \leq x/s \leq 0{\cdot}2$ is influenced by U'_∞/U_∞ and is increased by factors of about 2·6, 2·2, and 1·7 for U'_∞/U_∞ of 0·083, 0·057, and 0·042, respectively. The measurements also exhibit the marked fluctuations previously referred to, i.e., spots of higher heat transfer than the mean.

The heat-transfer measurements of McCormack *et al.* (**14**) for uniform-velocity flow in the range $5 \times 10^4 < Re_s < 8{\cdot}8 \times 10^4$ over a concave surface in the presence of vortices for $U'_\infty/U_\infty < 0{\cdot}01$ were also greater than those on an equivalent flat plate by an overall factor of about 2. There were, however, periodic spanwise variations between limits of 1·3 and 2·8 which corresponded with observed velocity variations caused by vortex motion. It has usually been found, e.g., Barcilon *et al.* (**31**), that vortex formation occurs for G_{δ_2} near the minimum possible value of 0·09, associated with which is the already-mentioned wave number of about 0·5, which remains substantially constant during subsequent amplification. It follows that the wavelength λ then increases with δ_2; over the blade pressure surface investigated by Martin *et al.* (**3**), λ would increase from about 0·87 mm when $G_{\delta_2} = 0{\cdot}09$ to about 2·08 mm at transition i.e., $x/s \simeq 0{\cdot}5$. Since there must always be an even number of paired opposed vortices covering a given blade span, increases in λ imply the progressive disappearance of end vortices in pairs, possibly by alternating oscillatory spanwise jumps. These will be reflected in transverse velocity fluctuations and must give rise to fluctuations in local heat-transfer measurements in the longitudinal direction at stations where vortices disappear because of a change in wavelength. Though this hypothesis seems to accord with observations, it clearly requires further experimental investigation.

To what extent Taylor–Goertler vorticity on the pressure surface influences empirical correlations for the length of the transition region, such as that of Dhawan and Narasimha (**5**), which is related to the onset of tran-

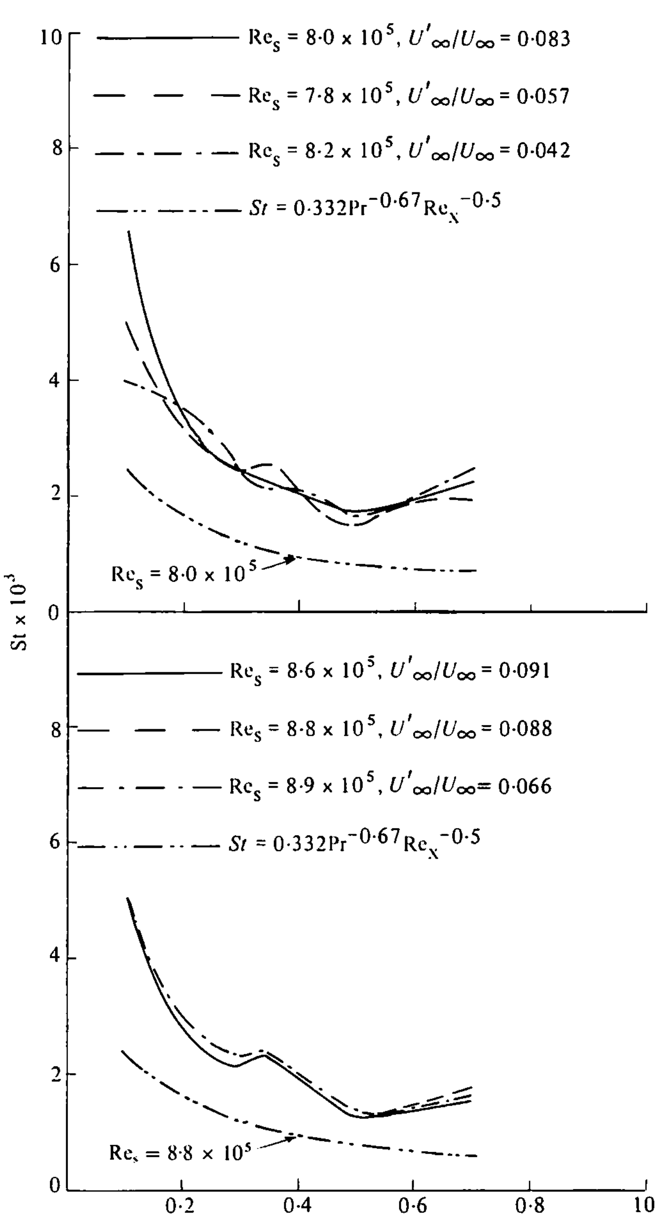

Fig. 8. Heat transfer measurements on the pressure surface of the blade of Martin *et al.* (**3**)

sition by a turbulence-spot intermittency factor, cannot yet be determined. Brown and Martin (**1**) found that for the aerodynamic conditions relating to Fig. 8, transition is predicted by this correlation to be complete near $x/s = 1{\cdot}0$, though the heat-transfer measurements extend only to $x/s = 0{\cdot}7$, at which station the heat transfer is up to one-third less than that for a fully-turbulent boundary layer. It is, however, noteworthy that the heat-transfer measurements downstream of the onset of transition do not exhibit the fluctuations discernible for $x/s < 0{\cdot}5$, suggesting that vorticity effects arising from wavelength changes are then less significant.

CONCLUSIONS

This study indicates that for a representative range of mainstream velocity distributions, Taylor–Goertler vortices will occur in the laminar boundary layer over the concave pressure surface of a turbine blade under engine operating conditions. This is so, not only for favourable pressure gradients typical of pressure surfaces, but also for zero and adverse pressure gradients up to the onset of laminar separation. Boundary-layer transition appears to be primarily controlled by the Launder (**25**, **26**) laminarization criterion on both suction and pressure surfaces, but whereas for the former for a given pressure gradient it is then essentially determined by U'_∞/U_∞ [according to the Seyb (**4**) or similar criteria], on pressure surfaces it is determined by interaction between Taylor–Goertler vorticity and mainstream turbulence, in terms of a transition Goertler number which diminishes with increasing U'_∞/U_∞. The observed multiplication of the laminar boundary-layer heat transfer and the fluctuations therein in the range $0{\cdot}2 \leq x/s \leq 0{\cdot}5$ on the pressure surface examined may reasonably be associated with amplification of longitudinal vortices and their irregular disappearance in pairs with increasing wavelength consequent on boundary-layer growth in the direction of mainstream flow.

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